COLLECTIVE DRAG EXPERIENCED BY AN ELECTRON

BEAM IN THE PURE RUNAWAY REGIME

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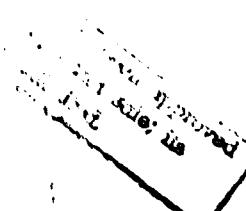
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ABSTRACT

A calculation is presented of the drag experienced by a continuously accelerated cold electron beam due to the swept excitation of the finite bandwidth two-stream instability.

Recently, it has been shown[1,2] that the momentum of a cold electron beam can remain constant (i.e., clamped) in the presence of an externally applied DC electric field E_o. The external push is offset by the continuous excitation of a collective mode supported by a suitable background structure (a plasma of density n_o or an equivalent slow wave structure) and travelling in synchronism with the beam. This clamping effect is associated with the formation of charge clumps[2] which are deeply trapped within the growing potential wells of the collective mode. The beam clamping regime appears when $E_o < E_T$, where E_T is the saturated amplitude of the cold beam-plasma instability. However, for $E_o > E_T$ (more precisely $E_o \gtrsim 2E_T$) the clamping effect is destroyed and the beam enters the pure runaway regime. In this regime the velocity v of the beam increases monotonically in time, i.e., $v = v(t) = v_o(1+\alpha t)$, where v_o is the initial velocity and $\alpha v_o = a$ is the constant acceleration of an electron of charge -e and mass m. To lowest order, $\alpha = eE_o/mv_o$.

In the pure runaway regime the amplitude $E(k,t)$ of a given Fourier mode of frequency ω does not attain a sufficiently large level to produce clamping because the resonance condition $kv = \omega$ is satisfied only for a short time. Nevertheless, since the coherency of the beam is not destroyed, the beam-plasma instability remains active and gives rise to the growth of a broad spectrum of waves. This note presents a simple calculation of the drag experienced by the beam due to the swept amplification of the background noise as the velocity of the beam crosses the two-stream instability threshold, i.e., a given wavenumber k falls within the unstable bandwidth for a finite length of time.

The form of the collective drag force can be easily identified from the exact law[1] of momentum conservation for the beam-wave system, i.e.,

$$\frac{d}{dt} (P_b + P_W) = en_b E_o \tag{1}$$

where P_b refers to the momentum density of a beam having a number density n_b, and

$$P_W = \int_{-\infty}^{\infty} dk \, k \, (\partial\varepsilon/\partial\omega)_k \, \frac{|E(k,t)|^2}{4\pi} \tag{2}$$

is the momentum density associated with the collective modes satisfying the linear dispersion relation $\varepsilon(k,\omega) = 0$. In the case of interest to propagation through plasmas $\omega \simeq \omega_p$, with ω_p representing the electron plasma frequency. Furthermore, for a fast beam $\partial\varepsilon/\partial\omega \simeq 2/\omega_p$, and is independent of k.

The corresponding drag force density is given by $F_d = -dp_W/dt$, hence

$$F_d = -(\pi\omega_p)^{-1} \int_{-\infty}^{\infty} dk \, k \, \gamma(k,t) |E(k,t)|^2 \tag{3}$$

where the time dependent growth rate γ enters through

$$E(k,t) = E(k,o) \, \exp \left[\int_0^t dt' \gamma(k,t') \right] \tag{4}$$

The principal feature of the effect under discussion is the bandwidth of the unstable spectrum, since the amount of momentum that a given mode can extract out of the beam depends on how long the mode remains unstable. The relevant time of interaction Δt is determined by the condition $a\Delta t = \Delta(\omega/k)$, where $\Delta(\omega/k)$ refers to the phase velocity bandwidth of the unstable modes. The unstable spectrum for a cold low density beam, i.e., $n \equiv n_b/2n_o \ll 1$ has its

maximum growth rate $\gamma_m = (0.866)\eta^{1/3}\omega_p$, located at a wavenumber $k_m = \omega_p/v$. The corresponding half-width of the unstable spectrum[3] is $\Delta k = (0.06)\eta^{1/3}k_m$. To incorporate this dependency into Eq (3), the time dependent growth rate is approximated by

$$\gamma(k,t) = \bar{\gamma}\left\{ H[k-k_-(t)] - H[k - k_+(t)] \right\} \tag{5}$$

where $k_\pm(t) = k_m(t) \pm \Delta k(t)/2 = k_0(1\pm\delta/2)/(1+\alpha t)$, $k_0 = \omega_p/v_0$, $\delta \approx (0.06)\eta^{1/3}$, and H refers to the Heaviside (step) function. In Eq (5) $\bar{\gamma}$ refers to a suitably averaged growth rate (e.g., $\bar{\gamma} = \gamma_m/2$ in its simplest form). An important simplifying feature of the beam-plasma instability contained in Eq (5) is that the magnitude of the growth rate, and the fractional bandwidth $\Delta k/k_m$ are both independent of the instantaneous velocity of the beam.

Utilizing Eq (5) in Eq (4) yields the shape of the spectrum at time t

$$|E(k,t)|^2 = |E(k,o)|^2 \begin{cases} 1 & k<k_- \\ \exp[(t - t_-)] & k_- < k < k_+ \\ \exp(2\bar{\gamma}k_0\delta/\alpha k) & k>k_+ \end{cases} \tag{6}$$

where $t_- = \alpha^{-1}[(k_0/k)(1-\delta/2)-1]$. This spectrum exhibits a characteristic log $|E|^2 \sim k^{-1}$ tail associated with the decrease in sweep time as k increases; a feature arising due to the absence of dispersion for fast electron plasma waves (i.e., $\omega \approx \omega_p$).

Using Eqs (5) and (6) in Eq (3) yields

$$F_d = -(\bar{\gamma}/\pi\omega_p) \int_{-k_m\delta/2}^{k_m\delta/2} d\kappa \, (\kappa+k_m)|E(\kappa+k_m,0)|^2 \exp\left\{ \frac{\beta}{1+\kappa/k_m}\left(\frac{\kappa}{k_m} + \frac{\delta}{2}\right)\right\}$$

(7)

where $\beta = 2\bar{\gamma}k_0/ak_m$. Defining $y = \kappa/k_m$, and realizing that $\delta \ll 1$, transforms Eq (7) into

$$F_d \simeq -(\bar{\gamma}/\pi\omega_p) \, k_m^2 \, |E(k_m,0)|^2 \, \exp\,(\beta\delta/2) \int_{-\delta/2}^{\delta/2} dy \, \exp\,(\beta y)$$

(8)

$$F_d = - (\bar{\gamma}/\pi\omega_p) \, k_m^2 \, |E(k_m,0)|^2 [\exp\,(\beta\delta) -1]\,/\beta$$

(9)

which is correct for small δ and arbitrary values of $\beta\delta$.

From Eq (9) it is found that the condition required for sustaining the free fall behavior (i.e., small drag) is $\beta\delta \ll 1$, which in terms of physical variables becomes

$$eE_o \gg (0.1)\eta^{2/3}\omega_p mv$$

(10)

This criterion is the collective mode analog of the Dreicer runaway condition[4] associated with single particle Coulomb collisions. When Eq (10) is satisfied, the electron beam is found in the pure runaway regime and experiences a collective drag given by

$$F_d = - (0.03)\pi^{-1}\eta^{2/3}\omega_p|E(\omega_p/v,0)|^2/v^2$$

(11)

If E_o is not large enough to satisfy Eq (10), the beam experiences a strong drag force given by

$$F_d = -[(0.1)\pi]^{-1} (a\omega_p^2/\eta^{1/3}v^3) |E(\omega_p/v,0)|^2 \exp[(0.1)\eta^{2/3}\omega_p v/a] \qquad (12)$$

which eventually brings the beam into the clamping regime discussed previously.[1,2]

It should be noted that the zero order spectrum appearing in Eqs (11) and (12) is to be taken from the appropriate description of the zero order plasma system, i.e., either from thermal equilibrium noise or from a dense soliton spectrum, as is more appropriate for the small k region of relevance to fast beams.

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